agreement with those obtained by Marxman et al.[2] for Plexiglass-O_2 system for the same duration of burning. However, the curve is flattened in case of 10 mm port diameter grain and a maximum regression rate is observed at $x/D_p \approx 3.5$ after which the regression rate is approximately constant. The curves for higher port diameters also show a similar trend.

The average regression rates for different port diameter grains are shown in Fig. 3. It is clearly indicated that the regression rate increases considerably as the L/D_p ratio increases. This is in agreement with the theoretical results,[2] where the fuel regression rate depends upon the oxidizer mass flux. For the same rate of oxidizer mass flow, the oxidizer mass flux decreases as the port area increases.

The fuel mass consumption rate for different port diameters have been presented in Fig. 4. The rate of fuel consumption is approximately constant for L/D_p ratio range of 16 to 48. This may be explained by the tendency of hybrid combustion in a stoichiometric ratio which depends upon the oxidizer mass flow rate. For higher port diameters, that is L/D_p ratio less than 16, the rate of heat transfer from the flame zone increases by increase of port surface area resulting in higher fuel mass consumption rate.

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Optimum Exhaust Velocity for Laser-Driven Rockets

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RECENT computer studies† of payload transfers from low Earth orbit to synchronous orbit and return via laser-driven rocket propulsion have shown that trip time reaches a minimum as specific impulse is varied. These minimums were obtained for constant exhaust-jet power and specified payload mass. The computations were based on the perigee-propulsion laser drive described in Ref. 1. The present Note shows that such minimums occur for all missions and that the optimum specific impulse is primarily determined by the mission difficulty. More generally, this optimum specific impulse maximizes payload kinetic energy achievable with a fixed jet power and propulsion time. These optimums differ from those obtained for low-thrust propulsion systems with on-board power.

The laser-driven rocket, in which remotely generated laser power is used to heat propellant,[2] belongs basically to the class of specific-impulse limited propulsion systems (Type I of Ref. 3) if difficult (high $-\Delta v$) missions are being considered. For more modest missions, however, specific impulses below the maximum achievable may well be optimum. The observed minimization of trip time as specific impulse is varied can be demonstrated from the basic rocket equation:

$$m_p/m_0 = 1 - e^{-\Delta v/v_j} \quad (1)$$

where Δv is the required velocity increment, determined by the mission, v_j is the exhaust jet velocity, m_p is propellant mass, and m_o is the initial mass. Equation (1) is valid for impulsive-thrust increments in gravitational fields or for any constant v_j thrust period in field-free space. For raising payloads in the Earth's gravitational field via laser-driven rockets, neither the impulsive-thrust nor the field free condition really applies very accurately, but the Δv in Eq. (1) can nevertheless be regarded as an effective mission-difficulty measure (of the order of 8 to 15 km/sec for the round trip to synchronous orbit). The total impulse needed for a given mission can increase by as much as a factor of two as thrust/mass ratio decreases.[4] But in the range of values possible for a particular propulsion system, the range of total impulse (and hence effective Δv) is quite small; hence Δv can be considered to be independent of v_j.

To express Eq. (1) in terms of propulsion time T_p and exhaust-jet power P_j we can write

$$m_o = m_{\text{pay}} + (1+k) m_p \quad (2)$$

$$m_p = \dot{m}_p T_p \quad (3)$$

$$\dot{m}_p = \frac{F}{v_j} = \frac{F v_j}{v_j^2} = \frac{2P_j}{v_j^2} \quad (4)$$

$$P_j = \tfrac{1}{2}\dot{m}_p v_j^2 = \tfrac{1}{2} F v_j \quad (5)$$

where m_{pay} is payload mass, $\dot{m}_p$ is propellant flow rate, F is thrust, and k is the ratio of propulsion system mass (including tankage) to propellant mass.

Introducing Eqs. (2-4) into Eq. (1) and solving for T_p yields

$$T_p = \frac{\alpha v_j^2}{2} \frac{1 - e^{-\Delta v/v_j}}{(1+k)e^{-\Delta v/v_j} - k} \quad (6)$$

where $\alpha = m_{\text{pay}/P_j}$. For $k \ll 1$ (a reasonable approximation) Eq. (6) becomes

$$T_p = \frac{\alpha(\Delta v)^2}{2} \left[\frac{v_j}{\Delta v}\right]^2 (e^{\Delta v/v_j} - 1) \quad (7)$$

Assume that α and Δv are independent of v_j; then differentiation of Eq. (7) yields the following result for minimizing T_p as function of v_j:

$$e^x(2-x) = 2 \quad (8)$$

where $x = \Delta v / v_{j,\text{opt}}$.

The solution of Eq. (8) (other than $x = 0$) is $x = 1.594$, or

$$v_{j,\text{opt}} = 0.627 \quad \Delta v \quad (9)$$

Thus the optimum v_j is directly proportional to Δv, and is independent of other variables. This result is simpler than that obtained for systems with onboard power sources (electric propulsion), for which optima depend on the specific mass of the power plant.[5] Using Eq. (9) in Eq. (7) yields:

$$T_{p,\min} = 0.773 \ \alpha(\Delta v)^2 \quad (10)$$

For the satellite raising mission ($\Delta v \approx 8$ to 15 km/sec), Eq. (9) yields optimum specific impulse in the range 500 to 1000 sec.

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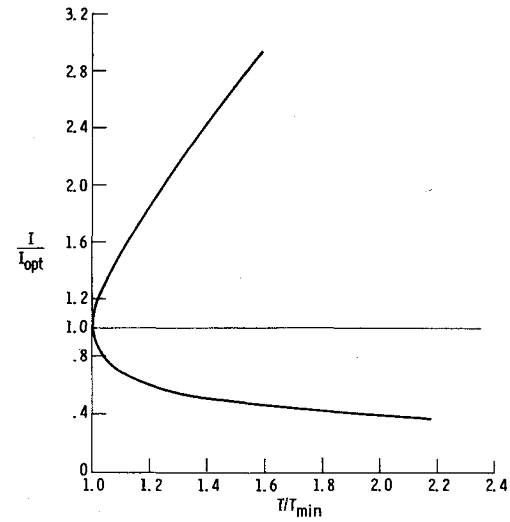

Fig. 1 Propulsion time as function of specific impulse [from Eq. (11)].

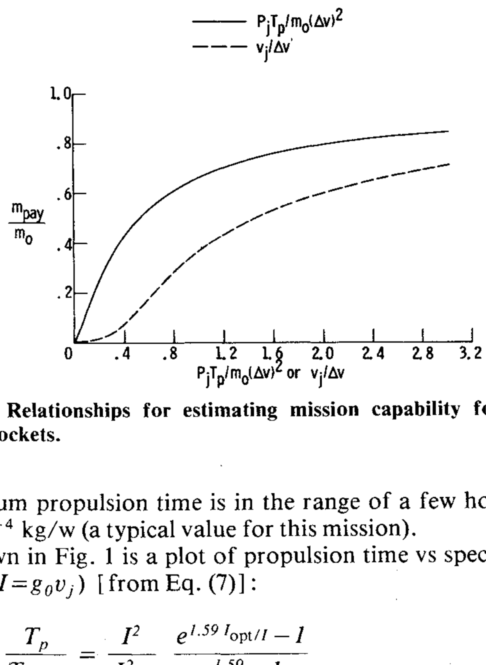

Fig. 2 Relationships for estimating mission capability for laser-driven rockets.

Minimum propulsion time is in the range of a few hours for $\alpha \approx 10^{-4}$ kg/w (a typical value for this mission).

Shown in Fig. 1 is a plot of propulsion time vs specific impulse ($I = g_0 v_j$) [from Eq. (7)]:

$$\frac{T_p}{T_{p,\min}} = \frac{I^2}{I^2_{\rm opt}} \frac{e^{1.59\, I_{\rm opt}/I} - 1}{e^{1.59} - 1} \tag{11}$$

This variation of T_p with I agrees qualitatively with numerical results.

The time-minimization shown in Fig. 1 results, as usual, from competing factors: For constant power, as v_j is reduced from high values the thrust increases [Eq. (5)] and thus the acceleration tends to increase and propulsion time goes down. However, the propellant mass, and hence total vehicle mass, increases as v_j is reduced, which tends to reduce acceleration and increase trip time. These competing effects due to reducing v_j produce the observed optima.

Inserting the optimum value of $\Delta v / v_j$ into Eq. (1) yields the result that, for minimum propulsion time on any mission with laser-driven rockets, about 80% of the initial mass is propellant mass and less than 20% is payload. Although this discussion was restricted to minimizing propulsion time, note that the parameter really minimized in Eq. (7) is $T_p / \alpha(\Delta v)^2$. Hence, for a given mission, the $v_{p,\rm opt}$ from Eq. (9) also maximizes the ratio $m_{\rm pay}/P_j$ for given T_p.

Even more generally, the quantity minimized by Eq. (9) is the exhaust-jet energy ($E_j = P_j T_p$) required to achieve a given payload energy ($E_{\rm pay} = (1/2) m_{\rm pay} (\Delta v)^2$). This accounts for the fact that Dipprey[6] recently obtained the same optimum $v_j / \Delta v$ (Eq. 9) by minimizing the amount of antimatter needed to achieve a given Δv for a given payload mass.

A similar derivation with m_0/P_j as a parameter instead of $m_{\rm pay}/P_j$ [using Eqs. (1, 3, and 4)] leads to

$$P_j T_p / m_0 = \tfrac{1}{2} v_j^2 \, (1 - e^{-\Delta v / v_j}) \tag{12}$$

which increases monotonically as v_j is decreased. Thus, for fixed initial mass, no minimum trip time occurs.

Using Eqs. (1, 2, and 12) (again with $k \ll 1$), the following relationship is derived between a propulsion time parameter and the payload ratio:

$$\frac{2P_j T_p}{m_0 (\Delta v)^2} = \frac{1 - m_{\rm pay}/m_0}{\ln^2 (m_0/m_{\rm pay})} \tag{13}$$

This relationship (shown in Fig. 2, together with the parameter $v_j/\Delta v$) is useful for estimating mission capabilities of laser-driven rockets.

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Reusable Multilayer Insulation Development

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Introduction

DEVELOPMENT of multilayer insulation (MLI) and its design is strongly dependent upon the environment in which the system must function. In recent years, much effort has been expended toward developing MLI materials and design concepts characterized by single use and moderate temperature environment requirements. The advent of reusable vehicles with the Space Shuttle development has imposed much more severe environmental requirements upon MLI systems. The MLI must be purged during ground hold conditions to remove condensible gases and repressurized during re-entry to neutralize the crushing atmospheric pressure loads. Provisions must be made to allow MLI venting during atmospheric ascent and to protect the MLI from repeated exposure to moisture and high temperatures during

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